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ZZT-domain Immiscibility of the Opening and Closing Phases of the LF GFM under Frame Length Variations

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Motivation and contribution

Current research has proposed a non-parametric speech waveform representation (rep) based on zeros of the z-transform (ZZT) [1]. Empirically, the ZZT rep has successfully been applied in discriminating the glottal and vocal tract components in pitch-synchronously windowed speech by using the unit circle (UC) as discriminant [1]. Further, similarity between ZZT reps of windowed speech, glottal flow waveforms, and waveforms of glottal flow opening and closing phases has been demonstrated [1]. Therefore, the underlying cause of the separation on either side of the UC can be analyzed via the individual ZZT reps of the opening and closing phase waveforms; the waveforms are generated by the LF glottal flow model (GFM) [1]. The present study demonstrates this cause and effect analytically and thereby supplements the previous empirical works; moreover, it demonstrates that immiscibility is periodically variant under changes in frame lengths; lengths that maximize or minimize immiscibility are presented.

LF glottal flow model (GFM)

Definition 1 LF glottal flow (derivative) model [2]

$$\begin{aligned} e_o(t) &= E_0 e^{\alpha t} \sin(\omega_g t), & t_0 \leq t \leq t_e \\ e_c(t) &= -\frac{E_e}{\epsilon t_a} (e^{-\epsilon(t-t_e)} - e^{-\epsilon(t_c-t_e)}), & t_e < t \leq t_c \\ e_s(t) &= 0, & t_c < t \leq T \end{aligned}$$

Let $e_o(t)$, $e_c(t)$ and $e_s(t)$ denote the opening, closing and shut phase respectively. The discretized equivalents of $e_o(t)$ and $e_c(t)$ are $eo = (eo_n)_{n=0}^{N-1}$ and $ec = (ec_n)_{n=0}^{N-1}$ respectively.

Zeros of the z-transform (ZZT)

Definition 2 Zeros of the z-transform

The zeros of the z-transform of a sequence $(x_n)_{n=0}^{N-1} \subset \mathbb{R}$ are defined as $z_1, z_2, \dots, z_m \in \mathbb{C} \setminus \{0\}$ such that $X(z_i) = \sum_{n=0}^{N-1} x_n z_i^{-n} = 0$ for $1 \leq i \leq m$.

The ZZT-transformation is denoted $\rho : \mathbb{R} \mapsto \mathbb{C}$, $\rho((x_n)_{n=1}^N) = (z_m)_{m=1}^{N-1-k}$, where x is a polynomial coefficient sequence ordered in descending powers, z is a sequence of non-zero zeros, and k is the multiplicity of a zero at zero.

References

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- [4] Q.I. Rahman and G. Schmeisser, *Analytic Theory of Polynomials*, Oxford University Press, 2002.
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Cauchy bound (CB)

Let $p(a, z)$ denote a univariate polynomial with variable $z \in \mathbb{C}$ and coefficients $(a_n)_{i=0}^{N-1} \subset \mathbb{R}$.

Theorem 1 Cauchy bound [3]

All zeros of a complex polynomial,

$$p(a, z) = z^n + \sum_{k=0}^{n-1} a_k z^k$$

lie in the disk $|z| < \lambda$ where $\lambda = 1 + \max_{0 \leq k \leq n-1} \{|a_k|\}$

Theorem 2 Cauchy bounded annulus [4]

Let $p(a, z)$ be a polynomial with zeros z_1, \dots, z_m ordered as $0 < |z_1| \leq \dots \leq |z_m|$. Let λ^* denote the CB of $p(a, z)$ and λ_* the CB of $z^m p(a, 1/z)$. Then the following inequalities hold,

$$\begin{aligned} \frac{1}{\lambda_*} \leq |z_1| \leq \frac{1}{(2^{1/m} - 1)\lambda_*} \quad \text{and} \\ (2^{1/m} - 1)\lambda^* \leq |z_m| \leq \lambda^* \end{aligned}$$

Thm. 3 and 1 are equivalent, but thm. 3 yield a tighter bound in the present analysis.

Theorem 3 Alternative Cauchy bound [5]

All zeros of a n 'th degree complex polynomial,

$$p(a, z) = z^n + \sum_{k=0}^{n-1} a_k z^k$$

lie in the disk $|z| \leq \lambda_a$ where

$$\lambda_a = \max \left\{ 1, \sum_{i=0}^{n-1} |a_i| \right\}$$

Subscript a denotes *alternative* CB.

Analysis of opening phase

ZZT representation of eo (cf. def. 1)

$$z_m = e^\alpha \rho(x_p), \quad z_m \neq 0, e^{\alpha \pm ik}, \quad m \in [1; N-2]$$

where

$$\begin{aligned} p(x_p, z) &= \sin(k)z^N - \sin(kN)z + \sin(k(N-1)), \\ k &= \omega_g = \pi/t_p \end{aligned}$$

Lower Cauchy bound of the ZZT rep.

If $\lambda_*^{-1}(N) > 1$ for the ZZT rep., all zeros lie outside the UC (cf. th. 2). As e^α is just a real scaling of the zeros of $p(x_p, z)$, $\lambda_*^{-1}(N)$ of $p(x_p, z)$ can be analysed in isolation heeding

$$e^{\alpha(h=1)} > (1/\lambda_*^{-1}(N)) \Leftrightarrow \alpha > \ln(\lambda_*(N))$$

Sampling period $h = 1$ (cf. ZZT rep. above).

The global minima points of $\lambda_*^{-1}(N)$ are

$$\left. \begin{aligned} \lim_{N \rightarrow a^\pm} \left| \frac{\sin(k)}{\sin(k(N-1))} \right| &= \infty \\ \lim_{N \rightarrow a^\pm} \left| \frac{\sin(kN)}{\sin(k(N-1))} \right| &= \infty \end{aligned} \right\} \Rightarrow \lim_{N \rightarrow a^\pm} \lambda_*^{-1}(N) = 0$$

where $a = (k + q\pi)/k = 1 + qt_p$, $q \in \mathbb{Z}$.

The global maxima points of $\lambda_*^{-1}(N)$ are

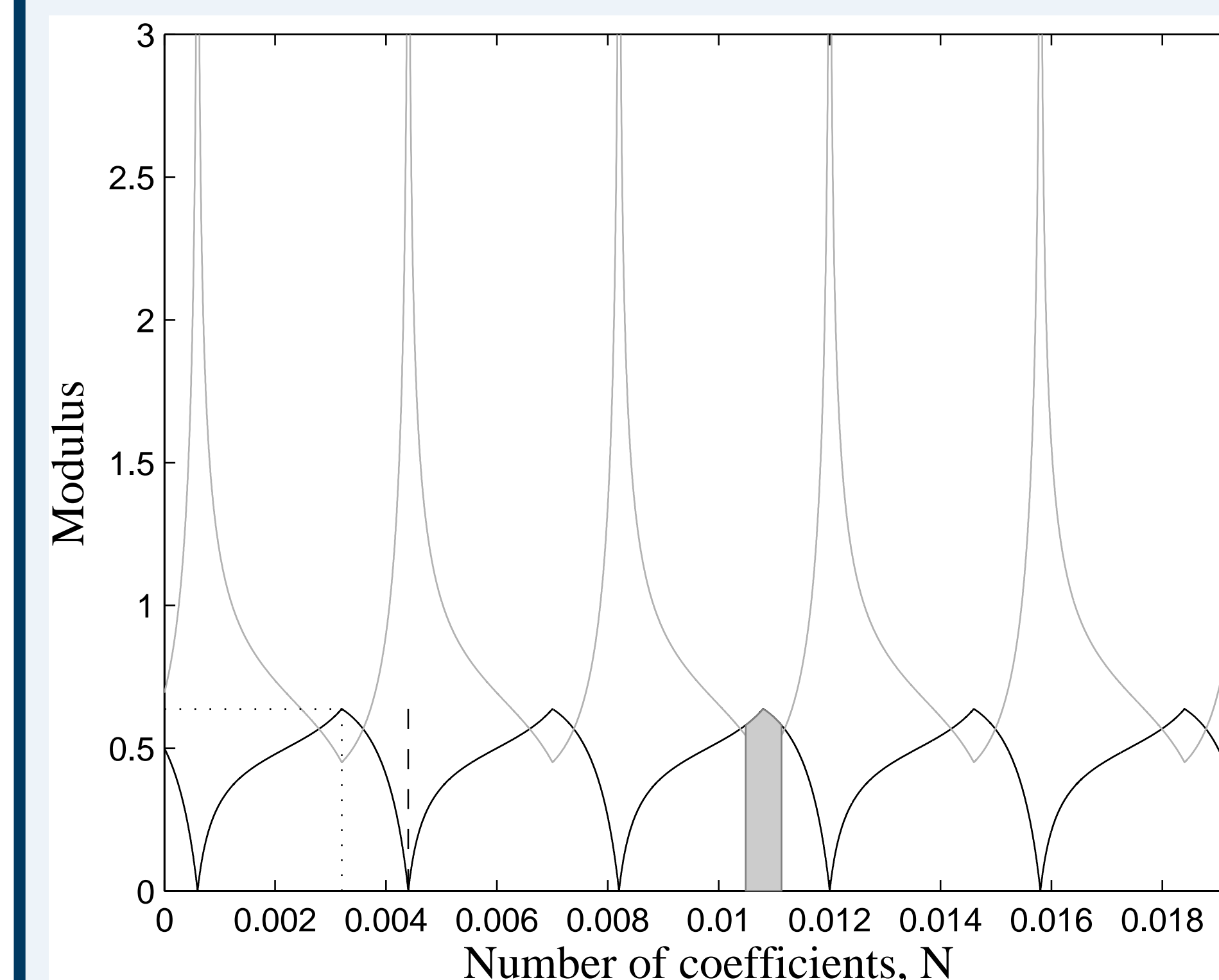
$$\left| \frac{\sin(k)}{\sin(k(N-1))} \right| = \left| \frac{\sin(kN)}{\sin(k(N-1))} \right| \Rightarrow \lambda_*^{-1}(N) = \frac{|2\cos(\pi/t_p)|}{|2\cos(\pi/t_p)|+1}$$

where $N = -1 + qt_p \vee N = t_p - 1 + qt_p$, $q \in \mathbb{Z}$

Numerical experiment

The LF GFM params. are set to common values,

$$\begin{aligned} t_0 &= 0.00000s & t_p &= 0.00380s & t_e &= 0.00480s \\ t_a &= 0.00031s & t_c &= 0.00800s & Ee &= 1.00000Pa \end{aligned}$$



Black line: Lower Cauchy bound. Vert. dashed line: A global min point. Vert. and horiz. dotted lines: A global max point and the max value respectively. Grey line: Lower bound of α . Grey region exemplifies a feasible neighbourhood for N . When $\lambda_*^{-1}(N) \rightarrow 0 \Rightarrow \alpha \rightarrow \infty$ why N must be chosen outside a neighbourhood of the global min points.

The illustrated feasible neighbourhood for N ,

$$\begin{aligned} \lambda_*^{-1}(N) &> (e^{-(\alpha=543.6428) \cdot (h=0.001)} \approx 0.58062) \Rightarrow \\ N &\in [-1 + qt_p - 82.68\%t_p; -1 + qt_p + 86.80\%t_p], \quad q \in \mathbb{Z} \end{aligned}$$

Analysis of closing phase

ZZT representation of ec (cf. def. 1)

$$z_m = \rho(x_p), \quad z_m \neq 0, 1, e^{-k}, \quad m \in [1; N-1]$$

where

$$\begin{aligned} p(x_p, z) &= (c_1 - c_2)z^{N+1} + (c_2e^{-k} - c_1)z^N + \\ &\quad (c_2 - c_1e^{-kN})z + (c_1e^{-kN} - c_2e^{-k}), \\ c_1 &= e^{\epsilon t_e}, \quad c_2 = e^{-\epsilon(t_c - t_e)}, \quad k = \epsilon \end{aligned}$$

Upper Cauchy bound of the ZZT rep.

If $\lambda_a(N) < 1$ for the ZZT rep., all zeros lie inside the UC (cf. th. 3).

The global minimum value of $\lambda_a(N)$ is

$$\lambda_a(N) = 1$$

which is achieved at

$$N = -\ln\left(\frac{1}{2} - \frac{1}{2}e^{-\epsilon} + e^{-\epsilon(t_c+1)}\right) / \epsilon \approx -\ln\left(\frac{1}{2}\right) / \epsilon$$

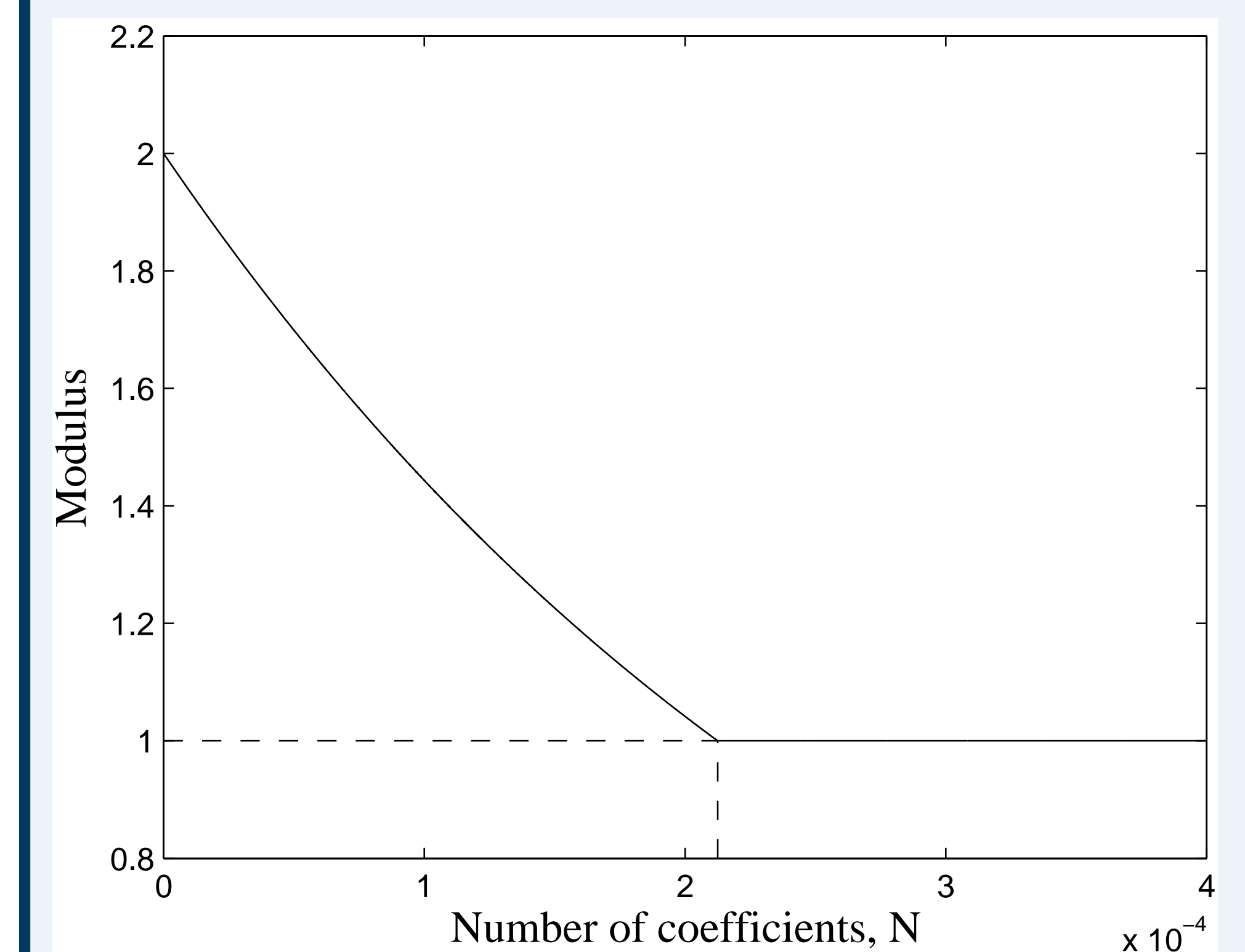
The global max point and value of $\lambda_a(N)$ are

$$\lambda_a(0) = e^{-\epsilon} + 1 + \frac{1 - e^{-\epsilon(t_c+1)}}{1 - e^{-\epsilon t_c}} \approx 2$$

Numerical experiment

The LF GFM params are the same as for the opening phase experiment. Further, ϵ is estimated iteratively by [2],

$$\epsilon t_a = 1 - e^{-\epsilon(t_c - t_e)} \Leftrightarrow \epsilon \approx 3261.44143$$



Solid line: Upper Cauchy bound of LF GFM closing phase. Vert. dashed line: A global min point. Horiz. dashed line: The global min value.

The global min point is reached at $N < t_c \ll 1$; thus, only the opening phase constraints on N must be considered when choosing a suitable sequence length.